Exercise 49

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = 2x^3 - 3x^2 - 12x + 1, \quad [-2, 3]$$

Solution

Take the derivative of the function.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(2x^3 - 3x^2 - 12x + 1) \\ &= 2(3x^2) - 3(2x) - 12(1) + 1(0) \\ &= 6x^2 - 6x - 12 \end{aligned}$$

Set $f'(x) = 0$ and solve for x.

$$6x^2 - 6x - 12 = 0$$

$$6(x^2 - x - 2) = 0$$

$$6(x - 2)(x + 1) = 0$$

$$x = \{-1, 2\}$$

$x = -1$ and $x = 2$ are within $[-2, 3]$, so evaluate f at these values.

$$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1 = 8 \qquad \text{(absolute maximum)}$$

$$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1 = -19 \qquad \text{(absolute minimum)}$$

Now evaluate the function at the endpoints of the interval.

$$f(-2) = 2(-2)^3 - 3(-2)^2 - 12(-2) + 1 = -3$$

$$f(3) = 2(3)^3 - 3(3)^2 - 12(3) + 1 = -8$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $[-2, 3]$.

The graph of the function below illustrates these results.